WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER ONE 2018
TEST 3: Vectors and Vector Calculus

Name: _____

Friday 11 May

Mark **/48 =** **%**

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet in **both** sections of the test.
- You are permitted one A4 page (one side) of notes in the calculator assumed section.

Calculator free section **Suggested time: ~25 minutes** **/22**

Question 1 (3 marks)

The vector form of a sphere is given by $\left|\underline{r} - \left(3\underline{i} - 2\underline{j} + 4\underline{k}\right)\right| = 6$.

a) Give the Cartesian equation of the sphere.

[1]

b) Does the point $(2, 1, -5)$ lie inside, outside or on the sphere? Justify your answer.

[2]

Question 2 (3 marks)

The points A, B and C have coordinates $(2, 1, -1)$, $(-2, 4, -2)$ and $(a, -5, 1)$ respectively, relative to the origin O, where $a \neq 10$.

Given $\overrightarrow{AB} \times \overrightarrow{AC} = (10 - a)\underline{j} + 3(10 - a)\underline{k}$ and the area of triangle ABC is $4\sqrt{10}$ square units, find the possible values of the constant a.

[3]

Question 3 (5 marks)

Solve the following system of equations by first forming an augmented matrix. Show each row operation beside the matrix.

$$x + y + z = 1$$

$$2x + y - z = -3$$

$$3x + 2y + z = 1$$

[5]

Question 4 (11 marks)

Referred to an origin O, the points A, B, C and D have coordinates $(1, 1, 0)$, $(3, 2, 5)$, $(0, -1, -4)$ and $(-2, -5, 0)$ respectively.

a) Find the vector equation of the plane $\prod$ passing through A, B and C.

[5]

The line l passes through D and is perpendicular to $\prod$.

b) State a vector equation of l.

[1]

The line l meets the plane $\prod$ at the point E.

c) Find the coordinates of E.

[3]

The point F is the reflection of D in $\prod$.

d) Find the coordinates of F.

[2]

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Calculator assumed section **Suggested time: ~25 minutes** **/26**

Question 5 (8 marks)

The planes $\prod_1$ and $\prod_2$ are defined by the equations $\underline{r}.\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} = 5$ and $x + 4y + z = -2$.

a) Find, to the nearest degree, the acute angle between $\prod_1$ and $\prod_2$.

[2]

The point A has coordinates $(2, 1, -2)$.

b) Find the perpendicular distance between A and $\prod_1$.

[3]

The plane $\prod_3$ is perpendicular to $\prod_1$ and $\prod_2$ and the point with coordinates $(0, 4, -1)$ lies on $\prod_3$.

c) Find the Cartesian equation of $\prod_3$.

[3]

Question 6 (6 marks)

A firework company is testing its new brand of firework. One of the company's employees lights the firework on a large area of horizontal ground and it takes off at a small angle to the vertical. After a flight lasting 8 seconds it lands at a distance of 24 metres from the launch point.

The employee models the firework as a particle and ignores air resistance and any loss of mass which the firework experiences.

Using this model, find for this flight of the firework:

a) the initial velocity vector

[3]

b) the initial speed, correct to 3 s.f.

[1]

c) the maximum height attained

[2]

Question 7 (4 marks)

Determine the point of intersection of the two lines below. Explain your working.

$$\underline{r} = \begin{pmatrix} 8 \\ -1 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix} \quad \text{and} \quad \underline{r} = \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

[4]

Question 8 (8 marks)

The position vector of a particle at time t seconds is given by $\underline{r} = (t^3 - 2t)\underline{i} + (t^2 - 3t)\underline{j}$.

a) Find the speed of the particle at time $t = 3$ seconds.

[3]

b) Does the particle ever come to a stop? If so, when and where does it stop? If not, explain why not.

[3]

c) Find the position vector when the particle is moving parallel to the horizontal axis.

[2]